HW1A. Written Homework 1A. **Due Tuesday, June 27, 2023 11:59PM**

Name:

Instructions: Upload a pdf of your submission to **Gradescope**. This worksheet is worth 20 points: up to 8 points will be awarded for accuracy of certain parts (to be determined after the due date) and up to 12 points will be awarded for completion of parts not graded by accuracy.

(1) Simplify the following expressions as much as you can. All figures are either vectors or scalars but multiple representations of vectors are used.

(a) $3\begin{pmatrix}1\\2\end{pmatrix} - 4\begin{pmatrix}4\\6\end{pmatrix}$

(b) $||(1,2,3)||$

(c) $||(-1,2,-4)||^2$

(d) $-3 + \begin{pmatrix}4\\-6\end{pmatrix} \cdot \begin{pmatrix}6\\-8\end{pmatrix}$

(e) $(5\mathbf{i} - 7\mathbf{j}) - (\mathbf{i} + 3\mathbf{k})$

(f) $\langle -1,2,0\rangle \times \langle 1,2,0\rangle$

(2) For the following pairs of vectors, either in $\mathbb{R}^2$ or $\mathbb{R}^3$, use the **dot product** to find the angle θ such that (1) θ represents the angle (in radians) between the two given vectors and (2) $\theta \in [0, 2\pi)$. Recall that for any angle θ, $\theta + 2k\pi$ for any $k \in \mathbb{Z}$ represents the same angle as θ.

(a) $(1,0)$ and $(0,1)$

(b) $(2,0,0)$ and $(5,1,0)$

(c) $\frac{1}{\sqrt{2}}(1,1)$ and $(\frac{5}{3},0)$

(3) Use the dot product calculation as done in Problem (2) to justify the following result/theorem. You may use properties of the dot product to simplify your calculations. (This problem will not be graded for accuracy.)

Let $\mathbf{a} = (a_1, a_2, a_3), \mathbf{b} = (b_1, b_2, b_3) \in \mathbb{R}^3$ such that the angle between them is given by $\theta \in [0, 2\pi)$. Then, the angle between $k_a\mathbf{a}$ and $k_b\mathbf{b}$ for any nonzero, positive scalars k_a, k_b is θ. i.e. the angle between vectors is invariant under scaling.

(4) Each set of points below represent vertices of a complex polygon in $\mathbb{R}^2$. Do the following: (1) draw the polygon on Cartesian plane and (2) compute the interior angles. Provide at least one dot product calculation. You may use geometric properties to simplify your calculations provided that you state them in your solution.

(a) $P_1 = (0, 0)$, $P_2 = (1, 0)$ and $P_3 = \frac{2}{\sqrt{2}}(1, 1)$.

(b) $P_1 = (0,0)$, $P_2 = \frac{1}{2}(\sqrt{3},1)$, $P_3 = (2,0)$, and $P_4 = \left(\frac{\sqrt{3}}{2}+2, \frac{1}{2}\right)$.

(5) Show the following pairs of vectors are orthogonal.

(a) $\mathbf{v_1} = (1,0,0)$ and $\mathbf{v_2} = (0,2,3)$.

(b) $\mathbf{w_1} = (\cos(\frac{\pi}{3}), \sin(\frac{\pi}{3}))$ and $\mathbf{w_2} = (\cos(\frac{5\pi}{6}), \sin(\frac{5\pi}{6}))$.

(6) For the following parts, find parametrizations of the line in the form $\mathbf{r}(t) = \mathbf{r_0} + t\mathbf{d}$ with $\mathbf{r_0}, \mathbf{d} \in \mathbb{R}^2$. Note there are an infinite number of valid parametrizations and you only need to provide one.

(a) The line passing through the point $(-1,1)$ with direction vector $\mathbf{d} = (1,1)$.

(b) The line passing through the points $(-3, 4)$ and $(4, -3)$.

(c) The line described by $y = \frac{3}{2}x + 4$.

(7) For the following parts, find an (implicit) equation describing the given plane in $\mathbb{R}^3$. Denote arbitrary points in $\mathbb{R}^3$ as (x, y, z) (i.e. as usual). Note there are an infinite number of valid equations and you only need to provide one.

(a) The plane passing through $(3, -4, 2)$ with normal vector $\mathbf{N} = (1, 2, 4)$.

(b) The plane passing through the points $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$.

(c) The yz-plane.

(8) Calculate the following projections.

(a) The projection of $(4, 3, 5)$ along $(1, 0, 0)$.

(b) The projection of $(\cos(\frac{\pi}{4}), \sin(\frac{\pi}{4})$ along $(\cos(\frac{3\pi}{4}), \sin(\frac{3\pi}{4}))$.

(c) The projection of $(1, 0)$ along $(4, 4)$.